Laser Interferometry of Parabolic Newtonian Telescope Mirrors with a Bath Interferometer and Analysis of Wavefront and Figure in OpenFringe.

V2.1

Michael S. Scherman © November 2012

Credits:
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Section I. Introduction

Interferometry: why even bother? For Newtonian telescope optics, the primary “focus” of this article, the parabolic primary mirror is of great importance. The concave parabolic indentation in the mirror’s disk brings to focus light of differing wavelengths in an apochromatic fashion. A spherical shape cannot do this as well, particularly if it is faster than f/10. Thus a way to determine how accurate the shape of the parabolic arc is, across its entirety from mirror edge to edge, is needed.

My sports car makes 585 horsepower. Ok, but a better way to say that would be that my car makes an average peak horsepower of 585 at 6,250 rpm on 88 octane unleaded pump gas off the flywheel at sea level, 80 degrees F with 52% relative humidity. This statement would be more revealing of the true nature of this vehicle. For a telescope owner, such as one who dropped over $3000-$4000 on a premium large aperture parabolic mirror (15 inches/38 cm or larger) for a newly built Newtonian telescope, an idea of the actual quality of the mirror is beneficial, more than for mere bragging rights. Consider said owner with temperature equilibrated, properly constructed, very well collimated telescope with a certified secondary mirror of known flatness, using premium eyepieces, on a night of supposedly very good seeing unable to really get a good focus of Jupiter at 200-300x. The great red spot is bland and contrast is poor. Edge sharpness is fuzzy. The Cassini division in Saturn’s rings is not very obvious, if at all seen. The second double of Mizar just does not resolve well as it should. Is it the seeing? Though the sky looks great, perhaps there are a lot of upper air disturbances. Or, is it the fact that the primary mirror has a problem such as a zone, astigmatism, spherical aberration, surface roughness, or a turned down edge. If so, how bad is it? Foucault and Ronchi are great tests for qualitatively analyzing such a mirror and these aberrations can instantly be seen. But how bad are they (as in enough to significantly affect the image)? This quantitation is therefore harder to accurately determine with these tests. Sure, a crude estimate of the Strehl ratio can be determined with Foucault, but it is in no way near as accurate as a test that measures many points across the entire surface of the mirror, a test such as laser interferometry.

The Strehl ratio (referred to as just “Strehl”) is a term for peak diffraction intensity and is calculated from RMS wavefront and is the measure of the fractional drop in the peak of the Airy disk as a function of wavefront error with a 1.0 Strehl being perfection. That is, a Strehl of 1.0 means 100% of a star’s light falls where it should, into the Airy disk. The Airy disk of unobstructed objective will contain a maximum of 83.8% of the energy from that star entering the objective, while the first order ring will contain 7.2% of the light, the second order ring 2.8%, and so on, diminishing with each successive ring. A Strehl of 0.9 indicates 90% of the light going to where it should in the Airy disk and the other 10% is going more into the first and second order rings of the Airy disk. A Strehl value of 0.82 or above (an RMS wavefront of 1/14.05 or 0.071 wave or λ), indicates the mirror
is diffraction limited (at the Rayleigh limit). Thus 82% of the light is going to where it should to make an image. Above 0.92 to 0.94 or so, and the mirror is optically very excellent (a premium hand figured and tested mirror by a seasoned pro usually falls in this category) and any improvements in image quality upon this will be hardly noticeable. You can brag about this one. Furthermore, it is nearly impossible to distinguish optical aberrations in a mirror with a Strehl of say, 0.99 from one with a Strehl of 0.97. A mirror with a 0.97 Strehl is $1/10^{th}$ wave P-V at the wavefront (1/32 or 0.031 RMS wavefront). Therefore, this is a mirror that would be as good as it really needs to be. A wave refers to the wavelength of the light source used to test the mirror. Given that the wavelength ($\lambda$) of a 587.6 nm laser is 5.876x10^{-7} meters or 2.313x10^{-5} inches) or less than half the size of a small bacterium, such a mirror with $1/10^{th}$ wave P-V at the wavefront (0.97 Strehl or 0.031 RMS wavefront) has deviations of no more than 2.94x10^{-5} mm over the entire optical surface. This is approaching molecular scale, as a hole or bump on the mirror surface is less than 2.94x10^{-5} mm deep or high. Incidentally, 2.94x10^{-5} mm is about 29 nm or 300 hydrogen atoms long!

These days some mirror maker/vendors are claiming wildly outrageous claims on some mirrors, even large aperture mirrors, with values such as 0.98 or 0.99 Strehl. Given it is difficult for even a top-notch optician to make a classic 6 inch (15 cm) f/8 mirror with a Strehl of 0.97, much less a larger aperture mirror, said claims are typically based on some “colorful” interpretations of a Foucault test. It is far more difficult to figure a large and fast mirror than a smaller and slower one, so such claims are to be met with a lot of skepticism. Worse, some mirrors are described with useless terms of “1/10 wave” or “1/10 wave P-V.”. When a mirror is described as $1/10^{th}$ wave, is that 1/10 wave at the mirror surface (surface accuracy), or is it 1/10 wave at the image plane (wavefront), or is it 1/10 wave entering the eye? A mirror with 1/10 wave surface accuracy actually is 1/5 wave at the wavefront, and if the vendor measures with just the half-wave amplitude, the mirror is only 2/5 (0.4) wave. I would be happier hear it is simply “diffraction limited”. P-V (peak to valley) is also a common metric, and it can be had with very few data points from one small section of the mirror, or if done right, from many readings all over a mirror’s surface. Therefore, Strehl (along with RMS wavefront) are the most useful “overall” values used to describe a mirror or lens optical quality. Moreover, RMS wavefront and Strehl derived from interferometry is by far the most accurate, as it involves many (hundreds to thousands) data points from the entire optical surface. Interferometry will even “see” aberrations evident in Ronchi and Foucault, such as surface roughness, zones, turned edges, and even scratches in the optical coating or the glass itself. Combine such an interferometric analysis with a nice Ronchigram and maybe a Foucault profile, and you have a complete and quantitative and qualitative picture of the mirror’s real figure.

Classic interferometry is different in nature. The light reflected off the test mirror (the test wavefront) is compared to the light reflected off a reference surface (the reference wavefront) of known quality. When the test and reference wavefronts are combined they form interference fringes (dark lines) whose form are an indication of the optical quality of the mirror under test. A telescope’s optical performance can be assessed by analyzing the degree of straightness of the fringes: the straighter the fringes, the higher the quality. An excellent and inexpensive interferometer option is the use of a Bath interferometer, typically of a right-angle type, as will be discussed. This design is a single path design using inexpensive optical components and is somewhat easy to fabricate.

The interference pattern seen using an interferometer, an interferogram (or “ingram” for short), contains quantitative information derived from a measure of the spacing of the fringes. When a telescope mirror is tested at the center of curvature (its radius of curvature, or ROC, which is twice the mirror’s focal length), one fringe spacing between fringe lines corresponds to 1/2 wave on the mirror’s surface. The wave referred to is the wavelength of the light source (laser in this case) you are measuring with. This is a microscopic distance at or near the molecular level. If a depression on the mirror’s surface causes a fringe to distort by one fringe spacing, the defect is 1/2 wave deep. This can actually be seen visually sometimes, as can a turned down edge, in the fringes of an interferogram, much like the bending of the lines in a Ronchigram.
To avoid confusion it should be remembered that an f/5 telescope mirror, when tested at the center of curvature, operates at f/10. Since telescope mirrors are generally checked at the center of curvature, when we consider an f/10 beam for testing purposes, it actually refers to an f/5 telescope mirror. Similarly an f/16 beam refers to an f/8 primary tested at the center of curvature. Mirrors are tested at their ROC, which can be either the center of a spherical mirror, or the local center (twice the focal length) of a parabolic mirror.

A drawback of interferometric (and other) testing is in the case of testing parabolic mirrors. The interferometric test introduces a spherical aberration (SA) due to the fact we are not working on a flat wavefront. This SA can be calculated with the radius, the radius of curvature (ROC), and the conic constant of the mirror, and then removed from the Zernike Polynomial(s). Nevertheless, if the mirror is large and/or very fast, this SA becomes important regarding the precision we are looking for (some L vs some L/100s). In that case, the test should be performed in autocollimation, or with a (expensive) nulling lens of very high and certified quality. These limitations are true for ALL testing methods (Fizeau, Bath, Wavefront analysis...). Fortunately there is another workaround. A numerical null is used to more easily see the error surface when we are trying to figure non-spherical surfaces. When testing paraboloids at the mirror’s ROC without nulling optics, the measured wavefront will have a non-zero Z8 component (principle spherical aberration): $Z8 = \frac{D^4}{(384 \times \text{Lambda}^2 \times R^3)}$

where $D$ = optical diameter, $R$ = paraxial radius of curvature, $\text{Lambda} = \text{test laser wavelength in nm}$. The desired amount of SA, known as Z8 for its Zernike coefficient number, is strongly dependent on the exact optical diameter of the mirror and the exact paraxial ROC of the mirror, since the SA goes as the 4th power of the diameter and inversely as the cube of the ROC. The paraxial ROC is the ROC measured from the center of the mirror, since the outside edges of the parabolic surface of the mirror focus light a bit farther from the center. Since the ROC is cubed in the formula, if it is measured inaccurately, the result will suffer an error in the numerical null that is 3 times larger than the error made when measuring it. For example, a 402 mm optical diameter, f/4.5 parabola mirror tested using a laser with 550 nm wavelength. The actual paraxial ROC was found to be 3616 mm and therefore $Z8 = 2.6 \times 10^{-6} \text{ m (2.6 waves)}$. $Z8$ (Wyant) is the 0 to peak amplitude, so the P-V would be 5.2 waves. Assuming you want the numerical null to be accurate to 0.05 waves P-V, then the SA calculation has to be accurate to one part in 104 (5.2/0.05), therefore the measurement of the paraxial ROC must be accurate to one part in 312, or about 11.6 mm. As the focal ratio drops this requirement becomes more stringent, so being able to measure the paraxial ROC to with 2-3 mm is ideal. Analysis software such as Openfringe or FringeXP uses this information to remove the desired correction from the data.

There are several ways to measure the paraxial ROC. One of the best is with a Foucault tester having a moving light source. Null the center of the mirror and measure the distance from the center of the mirror to the knife edge (KE). If the pinhole and the KE of the tester are at the same distance from the mirror, you have measured the paraxial ROC. If not, then take the average distance to the KE and to the pinhole. It’s easy to get within a couple of mm with this method. It helps to have an assistant hold the measuring tape. A laser range finder can work very well, but they are expensive and one must check the calibration under various operating conditions. Instead of the Foucault tester, you can also use a Ronchi tester. Again, null the center of the mirror with the edge of a single Ronchi line.

Considering these limitations, laser interferometry is still the best way to analyse a mirror’s figure and give quantitative as well as qualitative data. The Bath interferometer is a reproducible and accurate implementation of an inexpensive interferometer. The Bath interferometer was invented by Karl-Ludwig Bath in the 1970s and is derived from the Gates interferometer described in Malacara's book (Physical Optics & Light Measurement, Vol 26, Meth. of Exp. Phys.). The Bath interferometer differs from the Gates configuration by the addition of a small lens. Bath interferometers are quite easy to build. It is a common path interferometer that
uses a small lens as a reference surface generator. Since both the test and reference beams share a path through the air between the optics and the interferometer, some of the deleterious effects of bad seeing in this optical path can be avoided. W.H. Steel (Optica Acta, 1970, n°10, 721-724) noticed that in the common path interferometer "....both beams go through the system under test and vibration changes the two optical paths by the same amount." A collimated light source is divided by the beam splitter into the (blue) reference beam and the (red) test beam. The reference beam hits the mirror under test, reflects from this surface and passes through the lens. It comes to a focus at F3. The test beam is expanded into a spherical wave by the lens, which has a focus at F1. The expanding beam illuminates the mirror being tested and comes back to focus at F2. The two expanding beams pass back through the beam splitter and interfere at the detector. Because the focal locations F1 and F2 are separated laterally by the beam-to-beam distance, measurement astigmatism is inherent in the interferometer. For most measurement situations the astigmatism is small enough to be tolerated without correction. For those situations where the astigmatism is large enough to be bothersome, it can be calculated and removed from the wavefront error analysis.

The measurement-induced aberration is determined by calculating the difference in path length between the longest and shortest distances through the optical system for rays that originate at F1 and terminate at F2. For the case where a mirror is being tested at its center of curvature, the path-length difference, OPD, attributed to astigmatism is given by

$$OPD = \frac{D^2 d^2}{16R^3}$$

where

$D$ is the diameter of the mirror under test

$d$ is the beam-to-beam separation

$R$ is the radius of curvature of the mirror

$OPD$ is the path difference between the longest and shortest paths to the mirror that originate at F1 and terminate at F2

The following table gives the measurement-induced astigmatism for various beam-to-beam separations, when measuring a mirror at its center of curvature:

<table>
<thead>
<tr>
<th>Wavefront Optical Path Difference in Waves at 550 nm for a Selection of Mirror Diameters and Focal Ratios</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>6 mm beam separation</td>
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<tr>
<td>8 mm beam separation</td>
</tr>
<tr>
<td>10 mm beam separation</td>
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Fortunately, this astigmatism can be removed from an analysis with the Bath interferometer in programs such as Fringe XP or Openfringe when the test mirror is rotated and averages are taken of its optical surface in different rotations (to be discussed).
This diagram shows the right-angle version of the Bath interferometer. A collimated light source is divided by the beamsplitter into the (blue) reference beam and the (red) test beam. The reference beam hits the mirror under test, reflects from this surface and passes through the lens. It comes to a focus at F3. The test beam is expanded into a spherical wave by the lens, which has a focus at F1. The expanding beam illuminates the mirror being tested and comes back to focus at F2. The two expanding beams pass back through the beamsplitter and interfere at the detector (camera or projection screen). In practice, a complete optical system, not just a mirror, as in the example above, can be tested by the Bath.

Section II. Bath Interferometer Construction:

To construct a Bath interferometer, the best parts for the Bath interferometer include:

- 10-20 mm (15 mm is best for a wide range of mirrors) 50:50 cube Beamsplitter (coated or uncoated, although coated helps reduce ghost fringes) ideally made from BK7 optical glass. Some units are labeled as 1/10th wave or better surface accuracy which is good. Get a quality unit to ensure good glass melt.
- 12.5 mm flat front surface aluminum coated mirror, ideally 1/10th wave or better surface accuracy. A small triangular 12 mm prism also works well, assuming it is a quality unit.
- A precision made (research grade is best) 7.5 mm Biconvex lens (7.5 mm diameter), 10 mm focal length, made from optical quality BK7 glass is recommended. A lens between 5-10 mm diameter with a 5-20 mm focal length will suffice, as the focal length can vary somewhat as can the diameter. Be sure the lens has no bubbles in it from a poor glass melt during its manufacture. A 10-30X jeweler’s loupe can help to spot these.
- A quality LED laser module with an adjustable beam diameter (a 5-10 mm at 10 meters, divergence say, 0.1-2 mrad is good) with up to a maximum of 5-6 mw output and red (625-675 nm wavelength),
at 3 to 4 VDC is good. A 550 nm wavelength (green) is good too as that is the standard test wavelength (but may be annoyingly bright and can be hard to dim well). These modules typically draw <50 mA operating current. These LED laser modules usually come in handsome machined aluminum housings and have a glass lens (or a lens system). Helium Neon (HeNe) lasers cannot be used nearly as easily as they cannot be dimmed and their beams are usually too small to diverge enough to cover a mirror under test. It is a good idea to know the LED laser’s actual wavelength with some precision, as when testing parabolic mirrors, numerical nulling of a conic is desired, then the light source should have a well-defined, stable, and accurately known wavelength. Therefore a more expensive LED laser module with integrated driver and a removable/adjustable (focusable) glass lens using 3-4 VDC power is by far the best. These are designed to run continuously with a stable beam. Laser pointers can be used with somewhat less success than a quality module, although their duty cycle does not really support several hours of continuous runtime as well as a dedicated module. Many units such as those in cheaper laser pointers also have plastic lenses which usually have irregularities and bubbles in the lens, distorting the beam unnecessarily. It is a good idea to solder in new power leads to a laser powered by the expensive button type batteries, and running it off a larger power supply (like a regulated power supply as shown below or with AA, C, or D cell type batteries or even an AC adapter of the same specs) as imaging sessions can sometimes involve a few hours of laser time. A regulated power supply to safely power the laser module (even if it has its own integrated driver) off an AC adapter or other DC power source is shown below and highly recommended to ensure a stable beam (filtering to reduce power spikes). The voltage regulator allows different supply voltages to be supplied (6 to 12 VDC), making it easier to find a DC source. The 1-2 watt linear taper 1 K ohm potentiometer can be used to dim the LED laser to below its lasing threshold. Note: Avoid any laser over 5-6 mw output for safety reasons. A 5 mw laser briefly pulsed directly into the eye likely will not cause any lasting effects provided the blink reflex is fast, but a 100 mw laser can cause irreversible eye damage or blindness in an instant. Be sure to paint any shiny parts (nuts, bolts, metal lens mounts, etc) on the interferometer’s front flat white (or black) to minimize brilliant radiant scattering. White will help show the beam well, but black is even safer and will still show the return beams.

PJ 1: a power connector such as a center positive 5mm panel mount power jack.
SW1: a SPST 1-2A 125VAC SPST Mini toggle switch
C1: 22 µF 35 WVDC polarized electrolytic capacitor
C2: 0.1 µF 50 WVDC High K Dielectric capacitor
Pot1: 0.5-2 watt linear taper 1 K ohm potentiometer
R1: 10 Ohms 0.5W 5% Tolerance resistor
VR1: 7805 style 3.3 or 5VDC 1A Voltage Regulator (match to laser module max input voltage)
Laser 1: the LED laser module (may have its own integrated driver)
POT1 adjusts laser power both above and below the lasing threshold (below threshold and the beam dims precipitously and may become more round and “regular LED like”). C1 and C2 are surge filters, and C2 should be as close to Laser 2 as possible. R2 is a "safety" current limiter. Be sure to heat sink VR1, especially if input voltage differs from output voltage by more than 1-2 volts.

When building the Bath, it is best to fashion the mounting components out of aluminium using machine style screws for the adjustments. Aside from looking nice, aluminium is malleable and holds adjustments well. Again, paint the front surfaces of the interferometer (where the beams exit and return) flat white or black for safety. See diagrams below for spacing of the optical components, but be sure the distance between the laser beam leaving the diverging lens and the return beam that goes into the right-angle mirror must be kept to below 10 mm (5-6 mm is the best) to minimize off-axis false astigmatism (inherent to the Bath design). Moreover, the diverging lens must be as close to the beam splitter cube as possible, and as close to its edge to minimize vignetting.

As far as the 3 axis positioning stage for the Bath interferometer, height (up and down) is the Y axis. X is motion perpendicular to the laser beam axis (the optical axis) and Z in the forward-backward (focus-defocus; towards and away from the mirror) axis. X and Y translation are thus used for the tilts, and Z for the focus. Good sensitivity is needed for these (some µm). A scale on Z is useful for chromatism analysis. A stage with smooth height adjustment (Y axis) works well. The interferometer can then be mounted on an X-Z stage. The slide positioning brackets that have adjustment knobs from an old microscope work very well for this.

Below is the Bath in detail, notice the laser module positioning set screws (much like a telescope finder mount) and the clamps for the optics. Do not clamp the optics against bare metal, but add a piece of electrical tape as a pad between the metal of the clamp and the glass to prevent chipping. Place the beamsplitter cube with the splitting axis (the plane between the two sides) vertical in its clamp mount. During construction and alignment, keep the optics super clean, especially the laser’s lens, the beam splitter cube, the flat mirror (or prism) and the diverging lens.

It is a good idea to mount the diverging lens in an aluminium swing arm (see above as it appears in the interferometer, and below for the details) such that it can be flipped out of the way of the beam splitter if needed during alignment. To mount the delicate lens, drill a hole nearly through some 3-5 mm thick aluminium that is just smaller then outside diameter of the lens (7.0 mm in this case) and another hole nearly as deep that is a bit larger (7.5mm in this case). The lens will slide into the hole and rest against the flange made by the two different sized holes. Cut a thin piece of tubing 8 mm in diameter (in this case) into a tube -12 mm long and cut
out a relief. This is an expansion ring that will hold the lens in place. A small dab of adhesive can be used to secure it, being careful not to get on the lens. See illustrations below:

Biconvex Lens Mount

Side View (Cross Section)

Two forms of the diverging lens mount, the one on the right is better as it has an additional axis of movement and is easier to adjust. Position lens hole right at edge of mount to minimize beam separation.

The recommended positioning and spacing of the optical components in the right angle Bath interferometer are as follows:
The right angle Bath interferometer appears below with some positional adjustments to consider during alignment and operation. The large X, Y, and Z arrows indicate the movement of the entire interferometer during fringe acquisition as discussed. Again, focus is related to movement along the Z-optical axis (in line with laser beam). Tilt (X axis) is related to horizontal movement across the optical axis, Y is vertical. Adjustments are as follows:

**The Laser**: Rotation X for the beam to be center on the biconvex lens. Some ArcMin sensitivity. It is good to be able to rotate the laser module along the optical axis to increase fringe (rotate the module in its mount).

**Beamsplitter (cube)**: Horizontal rotation to remove secondary beams from the Ingram. 1° sensitivity. Place the beamsplitter cube with the splitting axis (the plane between the two sides) vertical in its mount clamp.

**Flat Mirror (or prism)**: Horizontal rotation to make both beams parallels. Some ArcMin sensitivity needed.

**Biconvex lens**: Height translation to center the laser beam, 1mm sensitivity. 90° rotation or the ability to remove the lens. Note that the cube and mirror settings may be fixed when correct. Take care of possible vignetting for very open beams.
When adjusting the angles of the optical components, the easiest way is to first be sure the laser beam goes through the beam splitter cube in its center (mid point of the cube’s top to bottom) and a 2-3 mm from the edge as shown in the figure above. If the laser module can be rotated about the laser beam axis and aimed with positioning set screws in its mount (see photos below of an example) it will make the setup of the Bath much better and easier.

With the interferometer body containing no optical components, first install the laser module in its mount. Place the interferometer on a table and measure the height of the laser module (where the beam would come out) from the floor. Position a center-marked square projection screen (around 25-30 cm on a side) at least 1-2 meters from the laser and measure the height to its center mark again from the floor. Switch the laser on and use the set screws to adjust its pitch such that were the beam exist the laser module and hits the center mark on the projection screen is the same height and horizontal. Be sure the laser (beam) is also 90 degrees perpendicular to where the optics and diverging lens will be. Holding a long meter stick parallel to the laser module body and verifying it hits a spot on the screen helps to accomplish this. This squares the laser beam (the optical axis) with the optics of the interferometer. The laser beam should he set such that it hits the center mark on the projection screen. Next install the diverging lens and verify the beam falls at or within 1 mm of the lens center (do not look into the lens with the laser on). Adjusting its height and angle may be necessary. In a darkened room the laser beam will now spread out in either a circular or rectangular shaped laser beam projection cone (depending on if the laser module has a circular or rectangular beam). It should be centered on the center mark of the screen as is the beam with-out the diverging lens in place. The laser module needs no further adjustment.

Roughly position the cube beam splitter and flat mirror (or prism) and swing (remove) the diverging lens out of place and activate the laser. Position the projection screen at least 30 cm from the diverging lens position (the front of the interferometer that is). Adjust the rotation of the beam splitter cube and the flat mirror (or prism) such the 2 small laser dots are very close to each other (within 10 mm, preferably 5-6 mm) right at the point they come out of the interferometer (a small piece of white paper held right at the face of the diverging lens can verify this). When satisfied, remove the small paper. Now make adjustments needed such that the two
beams are also other within 10 mm (preferably 5-6 mm) of each other on the screen. This will get the 2 beams as parallel possible.

Return the diverging lens to position and note on the projection screen the large circle of light from the diverging lens (or the large rectangle of light if the laser has a rectangular beam). Again this is the laser beam projection cone as it expands out of the diverging lens. The other small laser beam dot is the reference beam. Make sure the edges of the projected light cone (or the 4 edges of the rectangle if the laser’s beam is rectangular) are all equidistant from the center mark on the projection screen, and adjust the projection screen distance such that its far enough away that the laser beam projection cone nearly fills the projection screen. Hopefully the reference beam will be centered within the beam projection cone (centered, or to at least within 5-10 mm of the center mark of the projection screen). Here the expanding laser beam projection cone centers on the small reference beam. If needed, fine tune the flat mirror angle in its holder to accomplish this. Be sure illumination is even, otherwise further adjust the diverging lens position. The diverging lens now illuminates the mirror with the center of the laser beam projection cone (circle or rectangle). It also allows the most tilt in all directions. To check alignment, removing the diverging lens should give 2 beam dots very close together on a distant wall even at some distance of several meters (no more than 1-2 cm apart). Be sure the lens is fastened well back in place and secure the optics (verifying alignment s not lost) and the interferometer should require no further adjustments. The following figure Bath Interferometer Alignment below will help to visualize the final alignments.

A note about beam orientation: Both round and rectangular LED laser beams are polarized. The brightest and sharpest image is obtained when the long sides of the rectangular expanding laser beam projection cone are horizontal (providing the splitting surface inside the cube is vertical). Here it will be much easier to get wider spaced fringes (when fine tuning tilt for desired fringe pattern) as there will be a wider range of tilt and defocus. The downside is that the background of the expanding laser beam projection cone will be very bright such that the mirror disk may be hard to visualize (reduced contrast) and there may be a lot more beam artifacts in the image (blobs, bubbles, dark contrast-robbing patches, etc). When the long sides of the rectangular expanding laser beam projection cone are vertical (again, providing the splitting surface inside the cube is vertical), it will be more difficult to get wider spaced fringes (when fine tuning tilt for desired fringe pattern) as there will be a far narrower range of tilt and defocus. The upside is that the background of the expanding laser beam projection cone will be very faint and freer of beam artifacts and fringe contrast, though less will still be good and sharp. Having the beam at a 45 degree (or so) arrangement might be the best compromise. The same thing can be said for a circular beam, except that there is no way to orient the beam expansion cone, it is merely by rotating the laser module for desired contrast. The reason for this effect is that reflected to refracted beam intensity ratio depends on polarization direction of the input beam. Maximum fringe contrast is obtained when both beams have equal intensity. This is seen more when the rectangular expanding laser beam projection cone is somewhat vertical (for a rectangular beam). By rotating the laser module about its beam axis in its holder, beam intensity can be equalized and contrast maximized.
Bath Interferometer Alignment

Projection screen at least 30 cm from diverging lens (enough such laser beam projection cone fills screen:

Position reference laser beam dot at center or at least within 5-10 mm of center.

A finished example of the Bath interferometer (in combination with a Foucault/Ronchi tester) in the figures below. This design allows for the bath optical components to be fine-tuned and aligned and then locked down as described. The X and Z axis consist of the slide positioning clamp from a microscope and have fine enough movements with the knobs such that fringes can be obtained (micrometer sensitivity). It also has a nice Y axis elevation stage that has fine enough movement for getting good fringes (sub-millimeter sensitivity), and a Foucault/Ronchi tester built in for those type of tests. The whit box over the interferometer is windowed such the beams can pass through and keep the interferometer cleaner from dust. As it is white (matte) it helps when positioning the return beams during set-up for interferometry.
Section III. Laser Interferometry with the Bath Interferometer

Once the Bath interferometer is constructed and aligned, it can be used for interferometry. An analysis computer program such as Fringe XP or the newer and better Openfringe (based on Fringe XP) will be of huge help at this stage. This article will discuss analysis with Openfringe (current version 13.8). The best way to accurately analyze a mirror’s true figure by laser interferometry is by taking multiple images (interferograms or “ingrams”) of the temperature equilibrated mirror’s interference pattern (the ingram) in succession, analyzing each ingram to produce a wavefront file (such as an FFT wavefront file in Openfringe), and averaging these wavefront files together in Openfringe to eliminate artificial test system errors such as air turbulence in the optical path and fringe pattern changes. Then the mirror is rotated 90 degrees in the stand and again a succession of ingrams are taken at the next rotation. These are also averaged together for an average FFT wavefront file for that rotation. A total of 4 rotations of the mirror are taken, typically at 0°, 90°, 180° and 270°. In the end, the 4 averaged FFT wavefront files, each corresponding to a rotation, are then averaged together to give a final FFT wavefront file that is representative of the mirror’s true figure. The averaging of the 4 wavefront files together for the final FFT wavefront further eliminates test system induced errors such as trefoil and astigmatism from the mirror sagging under its own weight in the test stand, as well as astigmatism in the Bath Interferometer.

To begin testing, mark the mirror to orient it. As you are facing the mirror (optical side towards you), simply mark the 12 o’clock position on the mirror’s edge with a marker and write 0 degrees (0°) here. This is done on the edge of the mirror (not the optical surface obviously). Similarly, at the 6 o’clock position, mark the 180 degree position, and similarly mark the 90 degree position at the 9 o’clock position and the 270 degree position at the 3 o’clock position. It may be beneficial to mark the 0 degree position on the mirror optical surface with a small removable dot to aid in orientation of interferograms. It is important to be within 1 degree accuracy when marking the 4 positions of the mirror’s edge.

Next, determine where the mirror’s lateral center of gravity is, that is at what point on the edge of the mirror between the front face and back face the center of gravity (COG) is. A mirror that is laterally supported is how it will be tested and supporting the mirror laterally in the test stand (the mirror’s optical axis parallel to the ground -facing the horizon) means it is supported only by its edge (and not its back). This can induce a lot of astigmatism, even if supported in a sling. Say a mirror is 40 mm thick. Assuming it is indeed circular with a smooth edge and with parallel front and back faces that are flat and perpendicular to the edge, intuitively the center of gravity would be half-way, or 20 mm from the front face (and consequently 20 mm from the back face). Since the mirror has a parabolic indentation, it is actually shifted a bit more to the back side face. It is a good idea to draw a line all around the edge of the mirror at its lateral COG. Accuracy to within 1-2 mm of the lateral COG is essential. Thus lateral COG for a mirror is calculated as follows:

\[
\text{Sagitta} = \frac{\text{mirror diameter}}{16 \times \text{focal ratio}}
\]

To get the thickness of the mirror at its center “C” (versus the edge where it is thicker) and edge thickness (E):

\[
C = E - \text{sagitta}.
\]

Therefore, the lateral COG (again the distance from the back face of the proper distance in on the edge is)

\[
= \frac{(E^3 - C^3)}{3(E^2 - C^2)}
\]

So for a 16 inch (405 mm) f/4.5 40 mm thick mirror, its sagitta is 5.63 mm (the depth of the parabola at center). The position on the mirror’s edge of lateral COG is 18.6 mm in from the back face of the mirror.
The mirror can be best mounted in a test stand laterally either in a sling (sling method), consisting of a braided steel cable that supports the mirror over 180 degrees of its lower edge as it hangs, or with 2 supports disks (2 support method) such as metal roller bearings, 90 degrees apart (at 135 and 225 degrees) that support the mirror at its lateral center of gravity. Avoid slings that are wide or thick, and slings made out of rubber or nylon (such as straps like what car seatbelts are made of). Slings made of 1-5 mm thick braided steel cable (uncovered – bare cable), depending on the size of mirror, are best (a 4-5 mm thick steel cable will easily support even giant 100+kg mirrors). Be sure the cable is not so thick that the mirror’s weight is insufficient for it to hang down properly -contacting the cable through the lower 180 degrees of its circumference. Avoid support disks made of rubber or plastic, metal is best, such as roller bearings mounted on a thick long bolt to the test stand. Support disks ~0.5 cm thick and 2-4 cm in diameter suffice for most mirrors. For the cable sling, it must be exactly on the line of the lateral COG for the full 180 degrees it is contacting the mirror edge. The two bearing supports must also both sit on the lateral COG line exactly. Both of these support designs will generate some astigmatism and trefoil, but this can be easily removed in the analysis of rotated and averaged interferograms (to be discussed). This sling method is the best way to support the mirror (as likely this will be how it is laterally supported in the telescope) and any astigmatism seen in interferometry will be reproducible in the rotations and therefore easily averaged out in analysis. This two support method will produce some trefoil astigmatism at the contact points, but again, this will be reproducible and averaged out in analysis. The following figures illustrate the two preferred mounting methods.

**Sling Method** *(best - very minor astigmatism - mirror in cable sling using 1-4 mm thick braided uncoated steel cable)*

**Ideal:**
- Mirror properly hanging in cable at lateral center of gravity of mirror
- Side view

**Avoid:**
- Avoid mirror hanging with cable at an angle
- Support post

2 Support Method *(very good - some trefoil astigmatism - mirror rests on two thin metal support disks)*

**Ideal:**
- Mirror 90 degrees support
- 135° 180° 225°
- Side view

**Avoid:**
- Be sure to secure 0 degree position to prevent mirror from falling forward
- Avoid:
- Mirror >90 degrees support
- 180°
2 support method mirror support stands: Notice the safety restraint at the mirror’s 12 o’clock position. This can be removed (carefully) during interferometry provided the mirror does not tilt forward and fall out of the stand.

(photo credits: Berthold Hamburger and Dale Eason)
Previous figures: Sling method of mirror support stand: Again notice the safety restraint at the mirror’s 12 o’clock position. This one can be rotated out of the way for interferometry. Also notice the non-binding Velcro strips used to guide/keep the cable on the mirror’s lateral COG. Velcro is nice as the cable is not adhered to the edge of the mirror, it can freely slide under the Velcro, thereby minimizing binding astigmatism.

In the test stand, have a line at the 12 o’clock position on the test stand behind the mirror. Orient the 0 degree mark on the mirror edge such that they line up (to ensure accuracy to within 1 degree). This line will be used to match the marks at all rotations of the mirror. Be sure the mirror will not fall forward out of the stand by securing the top of the mirror (but not binding or putting any pressure on it).

Once the mirror is properly supported, some time, say an hour, should pass to allow it and the test environment to temperature equilibrate. The test environment should be free from any vibrations and air flow. A tunnel going from the interferometer to the mirror might be helpful to combat air currents as shown below. Heat from your hand when placed in the interferometer beam path will show nicely in the projected interferogram image as dark smoke rising from the dark silhouetted hand. Pretty cool, but exacerbates the need for a still air volume.

Figure credit: Jan van Gastel  http://members.ziggo.nl/jhm.vangastel/Astronomy/bath/bath.htm

Set up the Bath interferometer on its X, Y, Z positioning stage. Place the camera (a high resolution digital SLR camera is by far the best) on a height adjustable tripod with (ideally) a swing arm to allow the camera to be moved (swung) in and out of place in front of the beam splitter cube of the interferometer. A good positioning scheme with a digital SLR camera with zoom lens and a projection for visualizing the fringes (safely at high laser power) is shown below:
Place the interferometer with the diverging lens facing the mirror at the ROC of the mirror and turn it on, ramp up laser output such that it is lasing. DO NOT look directly into the ports of the interferometer or the return beams. Set up a flat-white projection screen as shown above a meter or so away from the out-port (looking into the beam splitter perpendicular to the laser optical axis) side of the interferometer and darken the room. Using the X, Y, and Z controls of the interferometer’s positioning stage will facilitate this. Aim the red dot in the center of the mirror, it does not have to be precisely in the center, but preferably within a cm or so. On the back wall behind the interferometer, look for the return beam dot (and a large diverged beam too). Center the small return beam on the diverging lens of the interferometer. The wide diverged beam on the wall will become a small dot at the mirror’s ROC. Be sure the small output beam of the interferometer is hitting at or near the center of the mirror and the diverging lens of the interferometer is at the mirror’s radius of curvature. Specifically, the return beams are likely projecting past the interferometer onto the wall behind. Adjust the angles of the mirror and interferometer such that the return beams fall on the front surface of the interferometer next to the diverging lens and mirror. If right at the center of curvature (the desired position), one return beam will be a few mm wide (round or rectangular depending on the shape of the source laser beam) and the other will be a tiny dot less than 1 mm wide. This dot will grow larger as one moves the
interferometer towards or away (focus and defocus) from the mirror (the Z axis), basically inside or outside the radius of curvature. The lateral side to side movement of the interferometer’s position (the X axis tilt) and its up-down position (the Y axis tilt) are helpful here to do this setup positioning. Be sure that the few mm wide return beam falls in the center of the diverging lens. The tiny 1 mm return beam should fall right into the interferometer’s flat mirror (and into its beam splitter cube). On the projection screen, a circular disk image of the mirror should appear, hopefully with red laser dots outside of it. There may be a large faint circle (or a faint broad bar if the laser source has rectangular beam) also nearby the mirror’s image. This is the diverged return beam image. Center this wide return beam over the projected image of the mirror on the projection screen. Fringe lines should then appear. As such, adjusting the X, Y, and Z positions of the interferometer starting with the diverging lens at the mirror’s radius of curvature is what is needed to generate fringes. Then employ a succession of tweaking the X, Y, and Z positions ever so slightly until a coarser (fewer fringe lines) fringe pattern develops. You should notice the center dot in the mirror (if it has one) ingram as well.

The following is a demonstration of what to look for while getting set up to take ingrams:

Here the optics in Bath interferometer are not properly aligned as the laser return beams are within the disk of the mirror (above figure). This will adversely affect FFT analysis (or fringe tracing) in Openfringe (discussed in upcoming sections) as these beams are in the fringes. In this example, those bright light dots are not coming from returned beams but from internal reflections in the beam splitter cube. This can be verified by blocking outgoing beams toward the mirror. Reflections will stay. These unavoidable reflections are not harming if interferometer components are adjusted so that reflections fall outside the fringe area (outside the edge of the mirror disk). Slightly adjusting the angle of the flat mirror (or prism) should move these beams just out of the disk and still maintain decent parallelism. Approximately 5 degrees of angle between the laser beam and normal to the splitter cube input facet helps as this changes the angle between camera optical axis and test beam cone axis. The fact the two beams are so close together is good, as that means the beams are rather parallel and the beamsplitter and flat mirror (or prism) are aligned well. What is also evident in this ingram image is the contrast ratio between the fringes and background is bad. Having a rectangular beam in the fully horizontal alignment (as described) may give an ingram like this. The ingram here actually has a good number of fringe lines at a nice orientation, however. Further adjustments will give fringe pattern as follows:
Starting at the radius of curvature, adjusting the defocus (moving forward or away from the radius of curvature-the Z axis) magnifies (right ingram) or demagnifies (left ingram) the fringes. It is a good idea to first get the fringes as large as you can (right ingram) and then fine tune to get more fringe lines. Very fine tilt and defocus adjustments will accomplish this. Here the circular fringes (the bull’s eye) are what are called closed fringes. These are not good for analysis, but basically show the interferometer is right on the optical axis. Moving off the optical axis very slightly will increase the linearity of the fringe lines as well as their number. Once satisfied with the large fringes in such as in the right figure, you can pretty much leave defocus (Z axis) alone at this point and simply adjust the X and Y tilts to move off the optical axis as shown in the next figures. The tradeoff is the lines will either become curved in the wrong direction or become more linear but too numerous and thin, thus losing contrast and sharpness. It will be far easier to get more and better linear fringe lines on slower (larger f/number) mirrors than faster ones. It is a matter of trial and error adjusting the axis.

Adjusting tilt in either Y or Z should help move off the optical axis where the fringe lines become more linear.
As you move off the optical axis, note the presence of linear fringes on both sides of the circular closed fringes (lower left side of the right Ingram). This indicates you are tilting in the correct direction. Continuing to adjust tilt should give more linear fringes that are thick, shaper, and numerous enough for analysis.

Tilt in wrong direction leads to circular fringes (note the circular portion on the right side of the disk in the left Ingram, as well as the curved nature of all the fringes). This will lead to a poor analysis. The right Ingram is much improved, however, as the fringe lines are straighter. This is what you would get tilting in the proper direction. Still there is too much fringe curvature and the circular fringe at the right side of the disk will lead to degradation of the analysis, as it will show up as a large peak or dip in the 3D surface profile during analysis.
The left ingram is improved and would be ok for fringe tracing. The circular fringe in the bottom left-center of the disk is still a bit too circular, however, and again, this area will show up in the 3D surface profile as a large spike or dip, thereby lowering the mirror’s figure values. Moreover, for FFT analysis, more fringe lines are needed and the ingram on the right is much better. The bubble in the middle should not affect analysis too much as its contrast ratio is much lower than the surrounding fringe lines. The circular components of the fringe lines are better minimized here in this figure too (notice the top fringe line). A defocused ingram is characterized by a poor edge and strange curving fringes near the edge. It is also better if the fringe lines are not too horizontal (as here) or vertical, but at an angle, such as 45 degrees (like the left ingram). This can improve Openfringe’s analysis when it picks fringe lines in FFT analysis.

The ingram on the right has too many fringe lines as well as being over exposed. Either decrease laser power or shorten exposure. Also, the consequence of too many fringe lines is that they fade into the background on the left and right sides of the disk in this ingram. This would be an unusable ingram, which when analyzed would give thousands of unwrap errors in FFT analysis. The ingram on the right has a good number of linear fringe lines.
and at a nice 45 degree angle. This would be good for FFT analysis. However, the ingram is also overexposed. Even though there are fewer and more contrasted fringe lines than in the left ingram, they also are degraded and broken up due to loss of contrast. This too would result in a lower than expected Strehl and RMS wavefront numbers as well as a large number of unwrap errors.

The ingram on the left is an excellent ingram for FFT analysis with minimized curved or circular fringes. Ingrams can be had with almost perfectly straight and well defined lines on slower mirrors, but for a fast mirror such as f/4.5, this is about as good as can be expected. This ingram is unfortunately a bit under exposed. Thus the fringe lines may be too rough. Either increase laser power or lengthen exposure. The ingram on the right has great exposure and though fewer fringe lines, is still good for FFT analysis. Notice the curved and wider spaced fringe lines in the left-center of the disk. Any more curved and spaced than this and the wavefront analysis could be degraded artificially. This might be about as good as one can get however with a really fast mirror such as an f/4.

Two good ingram examples, with a good number of high contrast and rather linear fringes all the way to the edge, far enough from optical axis (thus no closed [circular] fringes), and tilted at around 45 degrees. These were done on somewhat fast f4.5 optics too. This is what is needed for a good ingram. Unwrap errors on these
could be even well below 100 in Openfringe FFT analysis. The bubbles seen are inconvenient, but acceptable as they are not too dark in relation to the fringe lines. The two bright spots in the right ingram are acceptable as they may show up in the surface profile of analysis but can be removed with a low pass filter, as will be discussed.

Now that the fringe pattern and orientation is at its desired coarseness and angles, with more linear fringe lines, a single or multiple ingram images can be captured. A good ingram will have somewhere between 15 and 30 fringe lines if doing fringe tracing for analysis, however 25-50 lines is desired for FFT analysis. As will be discussed, FFT analysis is generally preferred as it is far more sensitive. It is better if the fringe lines are rather linear or mildly curved, but depending on the optic, that can be tricky to get. A fast mirror (less than f/5) will not produce as-straight fringes, but try and limit the number of heavily curved (circular component) or closed (circular) fringes. Again this is done by a combination of x/y tilt and Z defocus settings. More fringe lines improves surface analysis accuracy but only up to a point, as the lines become fainter and have less contrast if there are too many. Fewer than 15 and the figure of the mirror will not be as accurately represented over its entire surface. The sensitivity is related to the number of fringes. A small number of fringes are more sensitive to surface errors. However surface resolution is better with large number of fringes. Taken to the extreme one single fringe will tell you a lot about the area it covers but not about where it is not. Therefore a good compromise is somewhere between 30 and 40 fringes.

Check the projection screen to verify the desire fringe profile and be careful not to bump any component of the test environment (camera, interferometer, mirror, etc). Before beginning to take images, dim the laser down to below its lasing threshold, such that its beams are barely visible. The laser will look more like a bright LED than a laser if viewed directly (NEVER advised). Though dramatically dimmer, the ingram image will still be bright enough for direct eye viewing (seriously, NOT recommended) or photography. Ideally it is never a good idea to view the fringes with the eye, use the projection screen and then (after dimming) the camera’s view finder instead. It is better to be safe than sorry and is especially germane if you forgot to dim the laser before imaging and wish to avoid eye damage. When ready move the camera in to place and check the viewfinder for correct positioning of the mirror disk. The camera works best if mounted on a height adjustable tripod with a swing arm that allows the camera to be rotated (swung) into place a few mm in front of the beam-splitter output. When checking the fringes, swing the camera out of the way, tune up the laser and view the projection screen. When satisfied with the fringes, dim the laser and swing the camera into place (use its view finder to center the mirror’s disk with fringes in the view finder (and be aware of vignetting). It is now time to capture ingram images.

Capture multiple ingrams by photographing successively (or even multiple frames in a high resolution video) with the imaging camera in as close as possible to the output of the interferometer beam splitter cube (to reduce vignetting). With a digital camera, take the ingrams, separated by 10 seconds apart using a several second timer on the camera or a cable shutter release. Ten seconds between images is nice to be able to average out air currents over multiple ingrams. Moreover, be sure to slightly adjust defocus or tilt to change the angle of the fringe lines somewhat, every 2-3 ingrams taken, as one fringe angle may give a lower Strehl-RMS wavefront result than one that is slightly different, so averaging many such angle possibilities together helps to reduce such an error (in analysis). Hence, capturing 10-12 ingrams at each rotation is a good number to aim for. A 10 or more megapixel digital single lens reflex (DSLR) camera set to manual mode (M on the dial) with an aperture of around f/4.8 and a 1/30 to 1/40 second exposure works very well at ISO 640 using a 100 to 200 mm zoom lens set to 60mm (due to its wider field of view). Slower (like ¼ to 1/10 second at a lower ISO) will work fine too, but there may be more motion blur in the fringe lines. A lens should be used on the imaging camera to avoid diffraction issues. Be sure the mirror’s edge is in sharp focus in the viewfinder and in the resulting ingram (the fringe lines themselves will always be in focus as they are formed on the focus plane at each focus setting). An ingram at 60 mm zoom will look something like this:
Ingram from high resolution DSLR image  
Crop of ingram that Openfringe will likely produce

When this ingram is opened in Openfringe for analysis and the mirrors edge selected, it can be cropped to 640x640 pixels in the program for better analysis (discussed in an upcoming section). It is important that the camera has sufficient resolution (10 or more megapixels if the field of view is wide as in the example above) such that the crop of only the mirror’s disk is at least 640x640 pixels. Moreover, for good FFT analysis, the thickness of each fringe line in the ingram should span several (4-5) pixels. FFT analysis also works best if the fringe line itself is not too sharp and pixelated with a gradient of pixel darkness throughout the fringe lines thickness, that is, there is somewhat of a gradient of lighter pixels on the edge of the fringe line, and darker pixels in the center of the fringe line. To deal with this, in Openfringe go to Image Filter in the menu and select Blur in the drop down menu. The Blur Filter button will then be available. Then select the Blur Filter one or more times to slightly blur the fringe lines to give a better gradient as seen in the magnified box. In addition there is the fringe tracing filter that can be set up to 9. This filter will blur pixels together while tracing. In OpenFringe at least that filter value is a radius in pixels. So when you have a very high resolution image it is not as effective as it should be because 9 pixels may not be a very big part of the fringe. That is one reason auto tracing works better on low resolution images.

A camera with much less resolution must then have a magnified image of the mirrors disk such that it fills the camera’s field of view, again so that the mirror’s disk is at least 640 pixels wide and high. Incidentally the above ingram is 4900x3200 pixels, such that a crop of the mirror’s disk is 1300x1300 pixels, more than sufficient. Analyzing a 200x200 pixel crop of an ingram from a typical webcam for example, by resizing it up to 640x640 in Openfringe will introduce a lot of false error due to pixel artifacts from the resize (not enough pixels to span the thickness of the fringeline). Analyzing it at 200 x 200 pixels in Openfringe will work, but the surface accuracy will be much less as the fewer number of pixels will cover more surface area of the mirror, thereby decreasing resolution. Hence, standard VGA webcams and even somewhat higher resolution webcams are not advised for imaging. Large ingrams will also work in Openfringe where the mirror’s disk is larger than 640 pixels, but at a significant cost in speed. Therefore cropped to 640x640 pixels in Openfringe first works best and 640x640 is a very good compromise in size versus resolution. Consider that a mirror disk that is 620 or so real (not resized up) pixels or so across will give Openfringe some 301,700 pixels of data to derive a full mirror face wavefront from, far more than other tests methods such as multiple zone Foucault testing. The following figures illustrate these points:
The figure on the left is just barely adequate for FFT analysis, however, the fringe lines are too contrasted and are at the lower limit of pixels needed to span the fringe line. This is typical output one would expect with a web cam. The right figure is much better for FFT analysis. Here a high resolution DSLR (>10 megapixels) will give nice fringe lines with more than enough pixels spanning each line for accurate FFT analysis. This is helpful in some parts of an ingram where the lines become thinner too and there are fewer pixels spanning each line. Contrast and line sharpness, though lower here, is sufficient and allows the edges of the fringe lines to fade a bit. Thus, notice the darkness gradient of the fringe line, it is darker in the middle of the line and fainter towards the line’s edge. Openfringe FFT analysis uses this gradient to calculate the line’s true center in analysis, and this gives far more accurate results.

Be sure the camera and interferometer are kept perfectly still during the capture of multiple images, and the entire mirror disk is visible in the output beam (your camera’s viewfinder). When capturing ingram images, it is best to not to allow the illuminated mirror disk image to drift around the camera’s field of view too much while capturing images. The key is not to bump the interferometer. Record the ingram as either a low compression (very high quality) JPEG or ideally a lossless .png or .RAW file.

Ideally one should take between 2-12 ingrams of the mirror in the 0 degree position, again in the 90 degree position, again in the 180 degree position and finally again in the 270 degree position. Two ingrams are needed to make an average, at minimum, and more helps to increase the ability to average out errors, up to a point of diminishing return (time spent imaging for one thing). To get an idea of “seeing” at the test location (how stable the testing is and the amount of air turbulence), average 2 ingrams (as described in the following sections), then 3, then 4, etc and stop when the average values converge. Typically, for a test location with good seeing (very low air turbulence and vibrations where the fringe lines barely move) 4-5 ingrams should suffice at each rotation, and if you are really concerned about accuracy, take up to 10 or so. Again, for every few (2-3) ingrams taken, slightly adjust defocus or tilt to change the angle of the fringe lines somewhat (as one fringe angle may give a lower Strehl-RMS wavefront result than one that is slightly different, so averaging many such angle possibilities together helps to reduce such an error in analysis). Ten to twelve ingrams per rotation should suffice to be able to reduce turbulence and fringe orientation test aberrations during analysis.

To capture the ingrams at each rotation, orient the mirror in the test stand such that the 0 degree mark is at the 12 O’clock position right at the 12 o’clock mark on the test stand, fine-tune the positioning and check to be sure the fringes are as desired (as just described) and fine tune more as necessary. Let the mirror set in the test sling for at least 10-15 minutes as its figure will change as the glass sags in the new direction (you can actually watch the fringe pattern change as the glass settles, especially in a large thin mirror). Then, take the 2-
12 ingram images, and rotate the mirror’s disk clockwise \( \frac{1}{4} \) turn (the 90 degree position mark should now be in the 12 o’clock position, again align with test mark stand to within 1 degree) and repeat the ingram imaging. Do this for all four positions. The positions will rotate clockwise in the example ingrams as seen through the viewfinder of the camera as demonstrated below:

![Image of ingram images showing 0, 90, 180, and 270 degrees]

**Section IV. Analysis of interferograms in Openfringe**

To analyze an ingram, the program Openfringe (version 13.2 as of this writing) is a hugely powerful tool for analyzing single and multiple ingrams, as well as having the ability to manipulate and average them to reduce artificial errors. It is a hugely powerful interferogram analysis program besting some that cost thousands of dollars, and as of this writing is freeware. It is based on its powerful predecessor, FringeXP. To begin analysis of a mirror’s ingram image, open Openfringe and specify the mirror’s test parameters by clicking on **Configuration-Mirror/Test Parameters**. Give the mirror a useful ID, say “10 inch parabolic coated f5”. In the **Mirror** section, choose mm or inches and specify the mirror’s optical diameter to within 1 mm (that is the diameter only of the aluminized part of the mirror -not including the additional 1-2 mm from the edge bevel if there is one), its paraxial radius of curvature (the ROC measured from the center of the mirror), a central obstruction (not needed, but nice as that area will not be analyzed- this is usually the size of the minor axis of the Newtonian...
diagonal used with this mirror), and its conic constant. Since a numerical null is used for the parabola during analysis, the paraxial ROC must be measured accurately to within +/- 2-3 mm or better. If the test mirror is parabolic, the conic constant is -1.0, and if it is a spherical mirror the conic constant is 0. Next enter the Interferogram wave length in nm (the nm wavelength of the interferometer’s laser, hopefully this is known with some precision to +/- 1 nm). Since a parabolic mirror cannot really be tested at the radius of curvature accurately described, be sure that the Artificial Null check box is checked. A number like -2.124 or something should appear in the box beside. Fortunately Openfringe calculates this error and deals with this when determining the wavefront. As the bath interferometer is a single pass (common path) interferometer, be sure the Double Pass box is unchecked. The other fields can be left blank. Click on Save in a File button and save with a useful file name. Reopening Openfringe will load the last entered mirror data, so clicking on Read Existing File allows one to change mirror’s description without having to reenter the data. Next go to View in the menu and select Error Margins. Here in Null Variation, ROC and diameter tolerances can be set. Typically it can be very tough to measure the paraxial ROC down to 2-3 mm, so a value can be inputted such as +/- 2 mm or so. Select Compute to finish. Moreover, be sure that the checkbox in Configuration – Calculate Bath Astig – Remove Baths astig from analysis is unchecked at all times. Openfringe averages out constant test system errors such as Bath interferometer astigmatism and test stand astigmatism during the counter rotations and averaging of ingrams (as will be discussed). This is only a calculator to see how bad the Bath astigmatism you have is, quantitatively, but is already removed in the rotations, so again be sure it is not enabled at any time.

Other things to consider when setting up Openfringe are the settings for Zernike polynomials. When you change any of the Zernike terms in the dialog a new error surface will be displayed in the surface view. Select the surface view the select Menu – Zernikes -view terms from the Zernikes menu to open the Zernike terms Dialog window. To enable/disable a value click in the check box by that value. To change a value click on the value itself to the right of the check box. Then enter the new value. The value will be used as the coefficient for that Zernike term. The relationship of the value to the peak to valley or RMS error of that terms is complex and will not be discussed here. However you can switch to the profile view to see the exact effect. Several Zernike terms can be ignored:

Piston, Tilt, and Defocus - These terms are artifacts of the test system and not of the mirror itself. They are almost always turned off.

Coma - The coma term can depend upon the test system and the analysis technique. It can be ignored because it will be removed when the scope is collimated.

X Astig and Y Astig -This can be either test system induced or a defect on the mirror. Deselect these only if you know it is because of the mirror bending on the test stand (almost always the mirror will bend somewhat, so it is best to leave these turned off/unchecked during Ingram analysis). They should then usually be checked (turned on) after the final FFT wavefront file has been generated (with the 4 rotations averaged together), since presumably test stand astigmatism and sagging has been averaged out (if done properly) and the only remaining astigmatism is the actual astigmatism in the mirror. The purpose of Menu – Zernikes –view terms lets one turn X-Astig and Y-Astig on or off in the Zernike display. The purpose of Menu – Zernikes –Wavefront Zernikes selects what things are nulled out or removed from the wavefront. The default is to have these X-Astig and Y-Astig boxes unchecked (turned off) as it removes all astigmatism, system induced and the actual astigmatism in the mirror’s figure.

To analyze an Ingram open Openfringe and be sure the mirror’s test parameters are correct. Then go to File – Clear zerns and fringes to be sure all previous analysis of ingrams (Zernike files, etc) are cleared. Next select File – Open Interferogram and specify the Ingram image to open, then click Set Ellipse Points and place the cursor exactly at the left center (9 o’clock position) edge of the mirror disk image, click (a little white cross
appears at the edge of the mirror), and again on the opposite edge (the right center or 3 o’clock position). Notice the small magnified box to aid in exact positioning. Then click on Edge and Edge button and a white circle will appear circling the mirror. Use the arrow keys to center the circle on the mirror disk image and Ctrl + or Ctrl – to make the circle bigger or smaller. Then click the FFT Analysis Tab and re-center and resize more as needed such that the circle exactly circles the mirror disk down to 1 pixel. The following figures demonstrate this.

This is the preferred way to circle the mirror to get ready for FFT analysis. If however, the mirror (or the mirror’s image if using a camera with non-square pixels –geometric distortion) is not perfectly circular, then a multiple point selection of the mirror’s edge can be used to better outline the mirror’s edge. To do this, open the ingram, select Set Ellipse Points and select the first point as before. Then click on 4 or more equidistant points on the precise edge of the mirror, as shown below. Note the cursor position in the expanded view box on the right.
In either case, a white circle will appear more or less outlining the mirror’s edge. It should be right on the mirror edge all the way down to the pixel level with no areas protruding or intruding the white circle. This is important as a lot of false error can be generated if the mirror is not circled exactly. If not centered, center the white circle on the mirror image using the arrow keys on the keyboard. Selecting Shift + or Shift – allows the white circle to expand or shrink, aiding in properly circling the edge of the mirror. Next select the 1. FFT Analysis Tab and select the Center and Resize to 640x640 button. Here it is a good idea to further center and size the white circle on the edge of the mirror disk image in the ingram.

Once selected, it might be desirable to save the cropped ingram image and this can again be done by File- Save view as Image, with a useful file name like “90 degree crop”.

Now for the analysis. As described earlier, FFT analysis is preferred and much more accurate than fringe tracing. For FFT analysis, the number and shape of fringe lines in the ingram is important, as will be demonstrated below. Click the 2. FFT button to do the FFT analysis and the Fourier transform will display a dark box with some translucent white circles, something like in the figures below.
Ingrams of differing tilt and defocus with FFT analysis of 16 inch (40 cm) f4.5 parabolic mirror

A poor ingram with too few fringe lines with circular components

A good ingram will give a central bright spot (seen under the little dark disk), a fairly large clear area around the central spot, surrounded concentrically by either white dots or circles (these are called side lobes). Some side-lobes may be a series of concentric rings, much like the appearance of the Airy disk of a slightly out of focus bright star in a telescope at high magnification. A poor ingram will give a Fourier transform display with a smeary bunch of white blobs or something as side lobes. An ingram with too few fringe lines or fringe lines with too much circular components (curvature) in some of the fringes may give side lobes that are too close together (too close to the central spot). This will hugely affect the subsequent analysis of the mirror introducing false astigmatism and artificial surface profile warping. If the side lobes have more fainter side lobes farther out (see figures below), you may get some fringe print-through in the computed 3D surface of the mirror, where the
fringe lines themselves appear as ripples on the mirror’s computed figure, artificially distorting the results. Fringe print-through can be reduced through filtering in the upcoming analysis, so it is not a huge concern if it can be reduced to acceptable levels through filtering. Thus, choosing the number of fringe lines and minimizing circular components of fringe lines in an ingram is best to give a good FFT analysis as shown below.

Once satisfied with the FFT analysis side lobe appearance, continue the analysis. Over the bright central spot will be a dark disk. Expand the dark central disk until it comes very close to (but not touching) the side lobes, using the left and right arrow buttons next to the **Central Mask** window. This helps to suppress background illumination in the upcoming FFT analysis, giving a truer representation of the mirror’s smoothness and figure.
Or it might look something like this:

Beginning with this: Expand the disk until almost touching the side lobes

Next click the 3. **Compute Surface** button. Here Zernikes are computed and a surface plot of the mirror will appear. Look at the possible **Unwrap Errors** window and be sure it is below 1000, the lower the better (under 200 is really good). If not press the **Try to unwrap again** button to see if it goes down (it usually will not by much). A bad ingram that is unusable will result in unwrap errors of over 1000. Moreover, if a message box appears stating “the wavefront appears to be inverted, ok to be invert?” it is ok to click yes if the resulting conic constant (in the next screens that will appear) is at or near -1.0 (say -0.7 to -1.2). If it is near 0.0, then you don’t know which side the conic is on (so go back to the **FFT Analysis** tab, repeat **Compute Surface**, and do not invert, or simply click **Invert Wavefront**). To investigate the position of the conic, heat a portion of the optical surface, take an ingram and the heated area should show up as a hill. If it shows up as a dent, invert the wavefront under the FFT tab controls (**Invert Wavefront** button).

Next click on the **Surface** tab. Something like the following will appear. Here in this example, is a surface profile of the mirror and in this case it is more or less homogeneous, so this is an early indication that this is a fairly smooth mirror with a decent figure. The small cracks in the right hand side of the mirror’s disk (that are red) are the unwrap errors.
Here you can see the mirrors figure numbers like Strehl and RMS wavefront in the window header. Next click the 3D Button and a 3D plot of the FFT analysis will appear.

If there are any spikes (especially around the edge of the mirror like on the right side in the example above), click the Low Pass Filter window, and select a value of around 0.06 to 0.12 (0.06 is usually sufficient), and click apply. This reduces the aberrant noise of the spikes (reduce scatter) and the mirrors figure numbers improve somewhat. Generally it is recommended to always apply a low pass filter of 0.06 on any ingram. This filtering is not saved in the FFT wavefront file you save later on however, so it should be done each time you load a FFT wavefront file in Openfringe for analysis.
Next select the Zernike Based button under Surface Presentations and the following will appear.

The numbers for Strehl and RMS Wavefront can be noted and will usually improve. These are the values one records for a mirror’s final figure results (as discussed in following sections). To visually see any fringe print-through, surface roughness, ripple, zones, and turned edges, these may appear in the wire form diagram below:
Selecting the grid button in the menu gives an easier to visualize depiction:

In these examples, surface roughness is good and low (blue/green) and well below 0.06 waves. There is evidence of a turned up edge however (red and orange areas).
Selecting 0.06 for the low pass filter improves the smoothness (as well as the Strehl and RMS wavefront values).

To see various graphical reports, click the **Contour** button and the following appears:
Selecting the Profile button gives a graphical side-plot of the mirror figure as shown below, that can be rotated using the Angle circle. Notice the edge astigmatism on the right hand side of the graph in this example. This is likely due to test stand induced stress on the mirror (sagging in the sling). An upcoming section will demonstrate how this can be removed.

Finally clicking the Report button gives a nice graphical report with all of the previous screens in one view as shown below. Note: the results here are based off the FFT wavefront surface presentation values, but if you select the Zernike based button, they are based off the Zernike surface presentation.
Next Click File – Save FFT Wavefront to save this analysis with a useful file name like “90 degree-1”. Open the next ingram in this series (the next one for this rotation) and repeat the analysis process. You can also save the Zernike file File – Save Zernike file, if desired. In the end you will have multiple ingrams and FFT Wavefront files for this rotation. These FFT wavefront files can then be opened in Openfringe and averaged together to remove periodic errors such as air turbulence. Zernike files can also be averaged together, but doing this with FFT Wavefront files is easier and far more accurate. To do this, close and reopen Openfringe, and select File – Clear Zerns and Fringes, and then load all the FFT Wavefront files for this rotation you wish to average (File – Read FFT Wavefront). Control-clicking multiple files in the Open dialog box speeds file loading. Click on the WaveFront List tab and notice all of the loaded file names in the window box to the right of the 3D plot. Select the ones desired by clicking on the name (or click Select All if you wish to average all of them) to select the files to average and click Average.

An “Average” entry appears in the list box below the files loaded. Highlight it and select File – Save FFT Wavefront to save this analysis with a useful file name like “90 degree-Average”. This completes the generation of the average rotation file. Do this for all 4 rotations of the mirror such that in the end there are 4 average FFT Wavefront files (and 4 average Zernike files if desired), 1 for 0, 1 for 90, 1 for 180, and 1 for 270 degrees. Each of these files is an accurate representation of the mirror’s figure at that rotation with periodic errors such as air turbulence averaged out.

NOTE: Interestingly one can actually average the ingram image files themselves together to make a composite average ingram. This method is not advisable to use for multiple ingram analysis, as the average ingram can get fuzzy with a loss in contrast. To do such an average, again launch OpenFringe, then go to File- Average igrams. Select multiple images (the ingram images themselves) by Ctrl-clicking the images for each degree of rotation you want to select and click open. Average only the desired 0 degree images first, then the 90 degree images, etc. A progress bar will appear and when complete, click on the Interferogram tab. Hopefully a nice clean, tight image will appear of a good and high-contrast interferogram. If there is a faint (ghost ingram) mirror disk image that nearly superimposes on the main mirror disk image interferogram, or the interferogram is faint and washed out as shown below, this is due to the image moving in the camera’s field of view during image acquisition in
one of the selected images and will thus not make a good average interferogram. Redo with less images that are more similar and it should improve. If you are satisfied with the resulting average interferogram, save the image under **File – Save View as Image** with a useful file name like “90 degree average interferogram”.

Once all four average FFT wavefront files have been saved they can be averaged together in an average FFT wavefront File for the 4 rotations together. Again, this serves to average out test system errors such as test stand induced stresses (astigmatism) in the mirror, potato-chipping (a large thin mirror may sag a bit in the stand under its own weight deforming the parabola into a shape somewhat like a potato chip) or trefoil (mirror edge resting on a supporting peg will cause this). The analysis of this final average gives a final accurate indication of the mirror’s true figure.

First it is necessary to counter rotate each of the 3 rotations (90, 180, and 270 degrees) back to the 0 degree orientation (obviously the 0 degree does not need to be rotated). Again, as facing the mirror in this example, it was always rotated clockwise in 90 degree increments. Moreover, as this discussion utilizes a right angle Bath interferometer with the laser beam pointing at the mirror and the camera lens looking perpendicular to that axis into the interferometer’s starboard side, close and reopen Openfringe, and select **File – Clear Zerns and Fringes**, and then load the desired average FFT Wavefront file for this rotation you wish to average (**File – Read FFT Wavefront**), again the 90 degree average for this example. Next select **Transforms – Rotate Wavefront** and enter 90 in the **Rotate Angle** box, click **OK**. This (+ 90) is a counter clockwise rotation of 90 degrees. If however, the **Ingram Image** option box was checked **Flip left/right in Mirror Info** (under **Configuration**), then enter -90. Save the rotated FFT wavefront file with a useful name such as “Average 90 degree counter rotated”. Do the same for the other two rotations (for the 180, enter 180, and for the 270, enter 270). Save these FFT wavefront files as well.

Finally, reopen Openfringe, and select **File – Clear Zerns and Fringes**, and then load all 4 of the average counter rotated FFT Wavefront files (the 0 degree average, 90 degree counter rotated average, 180 degree counter rotated average, and the 270 degree counter rotated average) by **File – Read FFT Wavefront**. Again, control-clicking the multiple files in the Open dialog box speeds file loading if the files are in the same folder. Click on the **WaveFront List** tab and notice all of the loaded file names in the window box to the right of the 3D plot. Select the ones desired by clicking on the name (or simply click **Select All** since you wish to average all of them) to select the files to average and click **Average**. An “Average” entry appears in the list box below the files loaded. Highlight it and select **File – Save FFT Wavefront** to save this analysis with a useful file name like “Final 10 inch Mirror Analysis Average”. Therefore, the 4 average interferograms of the counter rotations averaged together give a final true representation of the figure of the mirror. A 3D representation of the mirror will appear. Click on the **Surface** tab, click **3D** button and apply the **Low Pass Filter** of 0.06.

Then note the Strehl and RMS wavefront values for FFT wavefront surface presentations in the 3D images header. Next select the **Zernike Based** under **Surface Presentations** option. Again, note the Strehl and RMS wavefront values (which may or may not improve, hopefully they are similar), along with Waves Best Conic. Click on the Report button and note the X Astig and Y Astig values under Zernike Terms section (in the Wavefront RMS column, not the Wyant column). Strehl and RMS Wavefront from FFT wavefront surface presentation are listed here as well. Write these values down or enter into a spreadsheet. These are typically the values one reports as the final specification of the mirror’s figure.

Another feature of Openfringe is that it can estimate the efficacy in the rotations/averaging methodology to reduce (average out) test stand induced artificial astigmatism. Selecting **File – Automation – Remove Test Stand Astigmatism**, allows one to select the FFT wavefront files for each rotation (that have NOT been counter rotated) and specify the counter rotation angles. Then **Run** is selected to generate average wavefronts and a PDF report that is informative as to whether stand induced astigmatism was removed by the
rotation and averaging methodology described herein. The PDF report describes what to look for in terms of success in removing false test stand induced astigmatism. The success of the Stand Removal depends upon having good averages at each rotation, knowing the rotation angle accurately to about 1 or 2 degree, and knowing that the test stand forces (stresses on the mirror as it hangs in the sling or as it sits on the pegs) are the same at all rotations. First, you can check how good the averages are by selecting File – Automation – Wave Map Stats. It is a little bit like the "Stand Removal" feature in that you input groups of wavefront files. It loads each one and calculates the astigmatism in each and then plots the groups on an astigmatism graph in a PDF file called astigPlot.pdf. Each group should be all the wavefront files from one mirror rotation that were used in the average, and many wavefront files can be selected at one time from that rotation. The Dialog for it will help through the process. The results should indicate how varied each analysis was. If they vary a lot then you need to average more data sets or find the cause of the variation and reduce it. Air currents are usually the biggest cause. The resulting graph has thick blue rings for the average at each rotation. Those averages will be similar to the ones in the middle page of the Stand Removal .pdf. There will be solid circles around the standard deviation of each rotation position. The smaller the circle, the better.

Section V. Results and what do they mean

As described, typically one reports the Strehl, waves best Conic, and RMS wavefront values after first applying a low pass filter of 0.06 (typically) on the final averaged mirror wavefront (after the rotations have been averaged) in both the FFT Wavefront surface presentation and after applying the Zernike Based surface presentation selection. These values hopefully will not differ much from each other. So it is best to record both surface presentation values in a spreadsheet or report for the mirror. Again, clicking on the Surface tab (if not already selected) and clicking on the Report button will show a nice combined report based off the FFT wavefront surface presentation or Zernike based surface presentation, depending on which one is selected. Sometimes the results in FFT wavefront surface presentation will be somewhat (or a lot) worse than the Zernike based surface presentation values. In that case, if the mirror actually has no major edge issues and is smooth and shows no serious ripple/roughness in FFT based wavefront surface presentation (and verified by Foucault/Ronchi), then the results in the Zernike based surface presentation can be reported as the final figure results (thus ignoring the FFT wavefront surface presentation values in the report) to simplify.

An important point to note is that the results in Openfringe, such as RMS wavefront and Strehl, are results based on a wavelength of 550 nm, a standard wavelength for reporting mirrors and other optics (and not the wavelength of the test laser in the interferometer you used). Openfringe automatically converts the interferometer test laser wavelength used to 550 nm when performing calculations and analysis. With the Strehl, RMS wavefront, and Waves Best Conic results from the Zernike based analysis (after the low pass filter was applied –see last section) from the final average of the mirror, these values can be used to ascertain how bad or good the mirror’s figure is. Remember, qualitative things that would be seen in a Foucault, Ronchi, or even a star test, such as surface roughness or turned edges/zones all contribute quantitatively to the wavefront as described by primarily by Strehl and RMS wavefront.

RMS wavefront and Strehl: These are the most important terms reported to describe the quality of the mirror’s figure. Again, a Strehl value of 0.82 or above (an RMS wavefront of 1/14.05 or 0.071), indicates the mirror is truly diffraction limited (at the Rayleigh limit), and therefore really good. This is about ¼ wave wavefront P.V. Thus 82% of the light is going to where it should to make an image. Above 0.92 or so, and the mirror is optically very excellent (a premium hand figured and tested mirror by a seasoned pro usually falls in this category).
Waves Best Conic: If the Waves Best Conic is fairly close to -1.0, that is in indication of a good figure for the mirror (correctly parabolic). A conic constant of -1.0 is a perfect parabola, 0.0 is a perfect sphere, and a conic constant of -0.5 and the mirror is half-way between a sphere and a parabola (and a lot of spherical aberration). Therefore a conic constant of -0.9 is a indicative of a slightly under corrected paraboloid, and one of -1.1 is indicative of a slightly over corrected paraboloid.

Astigmatism: Final astigmatism (or simply “astigmatism”) in the mirror is typically reported in waves RMS wavefront. As described, the report in Openfringe also reports astigmatism under Zernike as X Astig and Y Astig. To calculate the final mirror astigmatism, that is, the astigmatism in the mirror’s figure (and not from mounting it in the sling, as that was averaged out already):

Final Astigmatism in waves RMS wavefront = √((Xastig)² + (Yastig)²)

Microsoft Excel formula =SQRT(A1^2+B1^2)

where A1 = value for Xastig, and B1 = value for Yastig

Angle of Final Astigmatism in radians is calculated =0.5*ATAN(B1/A1)

To convert that to degrees:

angle of final astigmatism in degrees = angle of final astigmatism in radians*(180/π)

Spherical Aberration: the spherical aberration of the mirror is typically reported in waves RMS wavefront and can be calculated again in MS Excel from the conic constant supplied by OpenFringe:

Spherical aberration = 1/SQRT(5)*ABS(((A2+1)*B2^4)/(C2^3*384*(0.00055))

where A2 = inputted value of the conic constant (with its negative sign)
B2 = inputted value of the mirror’s optical diameter in mm
and C2 = inputted value of the mirror’s radius of curvature in mm

Table of Commonly Encountered Wavefront Relationships
(approximate Peak-Valley relationship to RMS wavefront and Strehl is derived from ¼ wave, P-V of spherical aberration, with wavefront equaling an RMS of 1/14.05 or 0.071).

<table>
<thead>
<tr>
<th>Approximate Peak-Valley Wavefront</th>
<th>Marechal RMS Wavefront</th>
<th>Strehl Ratio</th>
<th>Comments on quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/1 (2.0) “wave or λ”</td>
<td>1/1.70 (0.590) “wave or λ.”</td>
<td>0.050 *</td>
<td>Unusable-unfinished</td>
</tr>
<tr>
<td>1/1 (1.0)</td>
<td>1/3.38 (0.296)</td>
<td>0.090 *</td>
<td>Very Poor—but a good shaving mirror</td>
</tr>
<tr>
<td>1/2 (0.5)</td>
<td>1/6.71 (0.149)</td>
<td>0.390 *</td>
<td>Poor—good at lower powers only “light bucket”</td>
</tr>
<tr>
<td>1/3 (0.333)</td>
<td>1/10.60 (0.094)</td>
<td>0.710</td>
<td>Fair—good at medium to lower powers</td>
</tr>
<tr>
<td>1/4 (0.250)</td>
<td>1/14.05 (0.071)</td>
<td>0.820</td>
<td>Rayleigh Limit “Diffraction Limited”</td>
</tr>
<tr>
<td>1/5 (0.200)</td>
<td>1/17.54 (0.057)</td>
<td>0.880</td>
<td>Good—good at medium to higher powers</td>
</tr>
<tr>
<td>1/6 (0.167)</td>
<td>1/21.28 (0.047)</td>
<td>0.920</td>
<td>Very Good—a very pleasing optic</td>
</tr>
<tr>
<td>1/7 (0.143)</td>
<td>1/24.39 (0.041)</td>
<td>0.940</td>
<td>Very Good—good at high power</td>
</tr>
<tr>
<td>1/8 (0.125)</td>
<td>1/27.78 (0.036)</td>
<td>0.950</td>
<td>Excellent—great at very high power</td>
</tr>
<tr>
<td>1/9 (0.111)</td>
<td>1/31.25 (0.032)</td>
<td>0.960</td>
<td>Excellent - superb</td>
</tr>
<tr>
<td>1/10 (0.100)</td>
<td>1/35.71 (0.028)</td>
<td>0.969</td>
<td>Excellent —really, as good as it needs to be</td>
</tr>
<tr>
<td>1/11 (0.091)</td>
<td>1/38.46 (0.026)</td>
<td>0.974</td>
<td>Excellent —bragging rights- top 1%</td>
</tr>
<tr>
<td>1/12 (0.083)</td>
<td>1/41.47 (0.024)</td>
<td>0.978</td>
<td>Excellent —VERY FEW this good</td>
</tr>
</tbody>
</table>

* Low values of Strehl do not correlate well with image quality

For the average user, a mirror, especially a large aperture (15” -380 mm or larger) mirror, with a Strehl in the 0.5 to 0.6 range will provide satisfactory performance. While higher power planetary or double star viewing will certainly be less rewarding (loss of contrast and sharpness) than with a mirror with higher Strehl, lower power views should still be very pleasing. When the Strehl drops to around 0.4 does the image really
begin to suffer, even at low magnification (“low powers”). Low Strehl mirrors will not sustain 25-50x magnification per inch (10-20x per cm) aperture, however getting a superb quality (say Strehl 0.95 or above) 20 inch mirror to give a really good image of Saturn at 1000x under the best seeing on Earth is also difficult as the atmosphere limits the seeing and useful high magnification, no matter the telescope’s aperture. At the high end of optical quality, most people should be able to notice the subtle loss in low contrast detail and sharpness of a high contrast target like a planet in a telescope with a 0.98 Strehl primary mirror vs one with a 0.90 Strehl primary mirror, all things being equal. It is nearly impossible to notice image improvements when going from a Strehl of 0.97 to 0.99, so improvements above this yield hugely diminishing returns.

The Strehl ratio is derived from the RMS wavefront and is defined at each wavelength of light tested, and only measures optical performance at the center of the field of view. For example, a fast Newtonian reflector (f/4 or so) can have a Strehl of 0.98 at the center of the field, but 10 arc minutes away from the center of the field, the Strehl can fall below diffraction limited performance simply because of the coma present in such as fast system. Moreover, a mirror with a Strehl of 0.98 at 550 nm (green) can be sub 0.82 (~diffraction limit) at blue, yellow, or red wavelengths (this happens often). Therefore, the values of the quality of a mirror’s figure, such as RMS wavefront and Strehl, derived from interferometric measurements represents a ceiling on the attainable performance. In actual use, a mirror will never attain this performance, mostly because of mounting issues (mirror becoming asymmetrically supported as it is repositioned in the telescope when slewing from horizon to zenith, for example), the atmospheric seeing, collimation errors, and errors from the other optical elements in the path such as the secondary mirror. Even the steadiest nights at the best observation sites do not allow a system to achieve this performance, but it can come rather close.

If the values, such as Strehl/ RMS wavefront, you get for your mirror after analysis are less than expected, consider that a possible source of errors for your low number(s) is probably due to incorrect laser wavelength (the entered value in Configuration Mirror Test Parameters is incorrect). You therefore need to know the actual wavelength of the interferometer’s laser with some precision. Some laser vendors are not as precise and may even specify a range such as 670-675 nm. You can try different laser wavelengths in the analysis and see how that changes the analysis. If the mirror values improve, then the wavelength could be at fault. Another cause of low Strehl and RMS wavefront values is poor ingrams with heavily curved and circular fringes, too few fringe lines, or poor resolution fringe patterns (typically yielding high unwrap errors in FFT analysis). Avoid ingmas with unwrap errors over 1000. Yet another cause of low numbers is the mirror bending on the test stand and you have not yet mastered the technique for removing (averaging out) it as described in the previous sections. Be sure the mirror is properly supported and allowed to settle for some time before imaging. If there is a huge discrepancy between Strehl and RMS wavefront in FFT wavefront surface presentation and Zernike based surface presentations, this could be due to a turned edge, zone, or surface roughness (an actual defect in the mirror’s figure, and not a test-system error). In either the Zernike based or the FFT wavefront surface presentation, selecting several millimeters in the Ignore Outside Edge box can determine if the edge is contributing to a low Strehl/RMS (if it improves when ignoring the outer 1-2 cm of the mirror indicates a significant turned up or turned down edge). Finally FFT analysis can add high frequency noise to the analysis that looks like surface roughness and that two can reduce the Strehl. The application of the low pass filter helps a lot here (but is not saved in the FFT wavefront file, so it has to be reapplied when reopening a saved FFT wavefront file). Moreover, there are easier ways to determine mirror smoothness and edge condition than using interferometer. It can be done using interferometry but it takes a lot more work than simply using Foucault or Ronchi images. The two things that are hard to determine with interferometry are the mirror smoothness and edge condition. The qualitatively tell you a problem right away. Moreover, it is possible to estimate (albeit crudely) the RMS wavefront and Strehl from Foucault, assuming many readings are taken over the entire mirror’s surface through several rotations.
Case Study – reproducibility of the interferometric analysis:

A commercial made 16 inch f/4.5 coated parabolic BK7 glass mirror, 42 mm thick. Aluminum/SiO coating is in excellent condition with no scratches or worn areas. Manufacturer claims a 1/6 wave-diffraction limited figure (it is therefore assumed to mean 1/6 wave PV wavefront). Star test shows some astigmatism but good rings, and the mirror’s performance is very good in its telescope with excellent seeing for deep sky, however higher magnification views (>300x) suffer somewhat from softness. The Foucault/Ronchi patterns were good with somewhat minor apparent zones-roughness, but turned edges were noticed in the outer 1 cm, and mirror appears nicely parabolic with a somewhat strained surface profile.

Ronchigram: good parabolic, turned edge, off-center astigmatism/depression
Foucault at midzone, notice faint glass strain

Foucault at paraxial null (central zone)
At zone 8 (near edge) note turned edge
An 8 zone Foucault test done in all 4 rotations revealed an average Strehl of 0.629 with a P-V wavefront error of 1/3.013. The transverse error was 3.673 and the encircled energy ratio was 0.754. RMS surface error by this analysis was 28.3 nm.

For the interferometry, each analysis was performed on separate days and according to the methodology described. Each analysis consisted of 10-12 interferograms at 675 nm laser wavelength analyzed and averaged together in Openfringe (v13.1) for each of the 4 rotations at 0, 90, 180, and 270 degrees. Each rotation was then de-rotated and averaged to give a final FFT wavefront file. Mirror was tested at ROC and a 75 mm central obstruction was used in calculations, as was an artificial null of -2.131. For the final FFT wavefront report, X-astig and Y-astig in “Zernikes view terms” and “Enabled Zernike Term Nulls” were both disabled (unchecked).

Examples of ingrams from the 16 inch F/4.5 mirror at each rotation

![Typical 0 degree](image1)
![Typical 90 degree](image2)

![Typical 180 degree](image3)
![Typical 270 degree](image4)
Average FFT wavefront for each rotation (10 ingrams per rotation) - FFT wavefront surface presentation 1st analysis. Views shown are not counter rotated.

0 Degree          90 Degree

180 Degree        270 Degree

Notice the reddened areas of astigmatism rotating with the mirror. This is due to actual astigmatism in the mirror’s surface and not test stand induced.

Counter rotated (+/CCW counter rotations done) FFT wavefronts combined (averaged together) for final surface profile:

Here the reddened areas of astigmatism are still present. This is the average of the 4 rotations, so system induced astigmatism should be removed at this point. Automated analysis in Openfringe indicated that test stand astigmatism was sufficiently removed, so this is likely real astigmatism in the mirror. Likely the astigmatism is primarily from the slight turned down edge present.
### Computed Results from Openfringe.

<table>
<thead>
<tr>
<th></th>
<th>Strehl (FFT wavefront surface presentation)</th>
<th>Strehl Zernike based surface presentation</th>
<th>RMS wavefront * (Zernike based surface presentation)</th>
<th>RMS wavefront * (FFT wavefront surface presentation)</th>
<th>Waves Best Conic</th>
<th>Spherical Aberration *</th>
<th>Final Astigmatism * (angle of astigmatism = 0.632 rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1st analysis</strong></td>
<td>0.620</td>
<td>0.589</td>
<td>1/9.10 (0.110)</td>
<td>1/8.70 (0.115)</td>
<td>-0.978</td>
<td>1/26.54 (0.038)</td>
<td>1/9.75 (0.103)</td>
</tr>
<tr>
<td><strong>2nd analysis</strong></td>
<td>0.584</td>
<td>0.572</td>
<td>1/8.80 (0.114)</td>
<td>1/8.50 (0.118)</td>
<td>-0.979</td>
<td>1/25.74 (0.039)</td>
<td>1/9.52 (0.105)</td>
</tr>
<tr>
<td><strong>3rd analysis</strong></td>
<td>0.554</td>
<td>0.541</td>
<td>1/8.40 (0.119)</td>
<td>1/8.00 (0.125)</td>
<td>-0.981</td>
<td>1/30.33 (0.033)</td>
<td>1/8.69 (0.115)</td>
</tr>
<tr>
<td><strong>4th analysis</strong></td>
<td>0.646</td>
<td>0.633</td>
<td>1/9.70 (0.103)</td>
<td>1/9.20 (0.109)</td>
<td>-0.975</td>
<td>1/26.54 (0.038)</td>
<td>1/10.54 (0.095)</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.597</td>
<td>0.584</td>
<td>1/9.00 (0.11)</td>
<td>1/8.60 (0.116)</td>
<td>-0.978</td>
<td>1/27.29 (0.037)</td>
<td>1/9.66 (0.103)</td>
</tr>
<tr>
<td><strong>Analysis 1-4 recomputed together in Openfringe</strong></td>
<td>0.607</td>
<td>0.595</td>
<td>1/8.90 (0.112)</td>
<td>1/8.80 (0.114)</td>
<td>-0.971</td>
<td>1/29.29 (0.034)</td>
<td>1/9.73 (0.103)</td>
</tr>
<tr>
<td><strong>Standard Deviation from mean</strong></td>
<td>0.038</td>
<td>0.038</td>
<td>0.007</td>
<td>0.007</td>
<td>0.002</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td><strong>Standard Error of the Mean (SEM)</strong></td>
<td>0.019</td>
<td>0.019</td>
<td>0.003</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.004</td>
</tr>
</tbody>
</table>

* Values are in waves (\(\lambda\)) RMS Wavefront

Analysis 1-4 recomputed together in Openfringe is all FFT wavefront files for each rotation in analysis 1-4 taken and averaged together in Openfringe. This is even more accurate as it is the combination of separate tests with differing fringe patterns. This row is what would be reported as the mirror’s final values. Moreover, the averages (mean) computed in the table above are VERY close to the average one would get in the Analysis 1-4 recomputed together row.

It is really good that the FFT surface presentation and the Zernike based surface agree very closely with each other, indicating a successful analysis. Also, the 4 separate tests agree with each other as well, thus demonstrating the superb reproducibility of the proper Bath interferometric testing environment (also, note the low standard deviations in each parameter). Interestingly the Foucault quantitative data is also very close to these results as well (and visually confirmed by Ronchi somewhat), further confirming the true wavefront of this mirror and its overall figure. What is therefore evident is that Bath interferometry by this testing methodology, even on such a large and fast mirror, is a viable and reproducible method for accurately and reproducibly qualifying and quantifying the errors in a mirror’s (or other optic’s) optical figure. Hence, a final set of firm values from either the FFT wavefront based or the Zernike based surface presentation for this mirror can be converged upon and believed, as these results confirm original predictions for this mirror.
<table>
<thead>
<tr>
<th>MIRROR FINAL RESULTS at 550 nm</th>
<th>decimal</th>
<th>fraction</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Wavefront:</td>
<td>0.112</td>
<td>1 / 8.90</td>
<td>λ</td>
</tr>
<tr>
<td>Strehl:</td>
<td>0.607</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Approx P.V. Wavefront:</td>
<td>0.377</td>
<td>1 / 2.65</td>
<td>λ</td>
</tr>
<tr>
<td>Final Astigmatism</td>
<td>0.103</td>
<td>1 / 9.73</td>
<td>λ</td>
</tr>
<tr>
<td>Angle of Final Astigmatism:</td>
<td>36.22</td>
<td>NA</td>
<td>degrees</td>
</tr>
<tr>
<td>Waves Best Conic:</td>
<td>-0.971</td>
<td>NA</td>
<td>slightly undercorrected</td>
</tr>
<tr>
<td>Spherical Aberration</td>
<td>0.034</td>
<td>1 / 29.29</td>
<td>λ</td>
</tr>
</tbody>
</table>

where $\lambda$ is waves at the wavefront

OpenFringe converts laser wavelength to 550nm, a standard value for reporting

While this is a very good mirror for lower power deep sky observing and ok for medium power observing; overall a fairly decent performer, refiguring by a good optician would be recommended to bring out its best.